Calc BC 10.3 Polar Practice

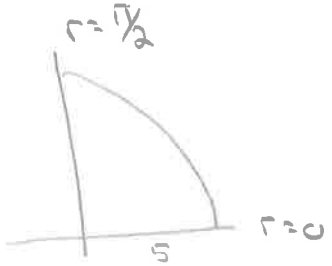
① $A = 2 \cdot \frac{1}{2}\int_0^{\pi/2} (3\sin\theta)^2\, d\theta$

$= 9\int_0^{\pi/2} \sin^2\theta\, d\theta$

$= 9\int_0^{\pi/2} \frac{1}{2}(1-\cos 2\theta)\, d\theta$

$= \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$= \frac{9}{2}\left(\frac{\pi}{2} + 0\right) = \frac{9\pi}{4}$

② 

Hey, this is a quarter circle w/ r = 5

$A = \frac{1}{4}\pi r^2 = \frac{25\pi}{4}$

③ $5\sin\theta = 2 + \sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$A = \frac{1}{2}\cdot 2\int_{\pi/6}^{\pi/2} \left[(5\sin\theta)^2 - (2+\sin\theta)^2\right] d\theta$

$= \int 25\sin^2\theta - 4 - 4\sin\theta - \sin^2\theta\, d\theta$

$= \int \left(24\cdot\frac{1}{2}(1-\cos 2x) - 4\sin\theta - 4\right) d\theta$

$= 12\left(\theta + \frac{1}{2}\sin 2x\right) - 4\cos\theta - 4\theta\Big]_{\pi/6}^{\pi/2}$

$\left(12\left(\frac{\pi}{2}+0\right) - 0 - 2\pi\right) - \left(12\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4}\right) - 2\sqrt{3} - \frac{2\pi}{3}\right) = \frac{8\pi}{3} + \sqrt{3}$

④ $y = \sqrt{3}x$ linear,

∴ Polar is of the form "$\theta =$"

$\theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\theta = \frac{\pi}{3}$

⑤ $x^2 + y^2 = 36$

$r^2 = 36$

$r = \pm 6$

⑥ $r^2 \cos 2\theta = 1$ $\quad \cos 2\theta = \cos^2 x - \sin^2 x$ $\quad x = r\cos\theta$, $y = r\sin\theta$

$r^2(\cos^2 x - \sin^2 x) = 1$

$(r\cdot\cos x)^2 - (r\sin x)^2 = 1$

$x^2 - y^2 = 1$

⑩ $r = 2(1+\cos\theta)$ $\quad L = \int_\alpha^\beta \sqrt{\left(\frac{dx}{d\theta}\right)^2 + \left(\frac{dy}{d\theta}\right)^2}\, d\theta$

$\frac{dx}{d\theta} = \frac{d}{d\theta}(2(1+\cos\theta)\cdot\cos\theta)$

$= \frac{d}{d\theta}(2\cos\theta + 2\cos^2\theta)$

$= -2\sin\theta - 4\cos\theta\sin\theta$

$\frac{dy}{d\theta} = \frac{d}{d\theta}(2(1+\cos\theta)\cdot\sin\theta)$

$= \frac{d}{d\theta}(2\sin\theta + 2\cos\theta\sin\theta)$

$= 2\cdot\cos\theta + 2\cos^2\theta - 2\sin^2\theta$

$L = 2\cdot\int_0^\pi \sqrt{\left(\frac{dx}{d\theta}\right)^2 \cdot \left(\frac{dy}{d\theta}\right)^2}$

$= 16$